4



# Transfer of Training in Problem Solving: A Final Report

Kenneth Kotovsky

Community College of Allegheny County & Carnegie Mellon University



This research was supported in part by the Personnel and Training Research Programs. Psychological Sciences Division, Office of Naval Research, under contract N00014-85-K-0696. Reproduction in whole or in part is permitted for any purpose of the United States Government.

SECURITY CLASSIFICATION OF THIS PAGE

# REPORT DOCUMENTATION PAGE

**1a. REPORT SECURITY CLASSIFICATION**
unclassified

**1b. RESTRICTIVE MARKINGS**

**2a. SECURITY CLASSIFICATION AUTHORITY**

**2b. DECLASSIFICATION / DOWNGRADING SCHEDULE**

**3. DISTRIBUTION / AVAILABILITY OF REPORT**
Approved for public release;
Distribution unlimited

**4. PERFORMING ORGANIZATION REPORT NUMBER(S)**
ONR-88-3

**5. MONITORING ORGANIZATION REPORT NUMBER(S)**

**6a. NAME OF PERFORMING ORGANIZATION**
Community College of
Allegheny County

**6b. OFFICE SYMBOL** *(If applicable)*

**7a. NAME OF MONITORING ORGANIZATION**
Personnel & Training Research Programs
Office of Naval Research (Code 1142PT)

**6c. ADDRESS** *(City, State, and ZIP Code)*
Behavioral Sciences Department
808 Ridge Avenue
Pittsburgh, PA 15212

**7b. ADDRESS** *(City, State, and ZIP Code)*
300 North Quincy Street
Arlington, VA 22217

**8a. NAME OF FUNDING / SPONSORING ORGANIZATION**

**8b. OFFICE SYMBOL** *(If applicable)*

**9. PROCUREMENT INSTRUMENT IDENTIFICATION NUMBER**
N00014-85-K-0696

**8c. ADDRESS** *(City, State, and ZIP Code)*

**10. SOURCE OF FUNDING NUMBERS**

| PROGRAM ELEMENT NO. | PROJECT NO. | TASK NO. | WORK UNIT ACCESSION NO |
|---|---|---|---|
| 61153N | RR04206 | RR04206-0C | NR702-011 |

**11. TITLE** *(Include Security Classification)*
Transfer of Training Problem Solving: A Final Report

**12. PERSONAL AUTHOR(S)**
Kenneth Kotovsky

| 13a. TYPE OF REPORT | 13b. TIME COVERED | 14. DATE OF REPORT (Year, Month, Day) | 15. PAGE COUNT |
|---|---|---|---|
| Final | FROM ________ TO ________ | October, 1988 | 11 |

**16. SUPPLEMENTARY NOTATION**

**17. COSATI CODES**

| FIELD | GROUP | SUB-GROUP |
|---|---|---|
| | | |
| | | |

**18. SUBJECT TERMS** *(Continue on reverse if necessary and identify by block number)*
Problem Solving, Transfer, Representation, Analogy. (SDW)

**19. ABSTRACT** *(Continue on reverse if necessary and identify by block number)*

This report summarizes the results of experiments that attempt to delineate problem characteristics that control transfer of training between problems, and to discover those characteristics that make problems hard or easy to solve. The problems used were sets of isomorphs of the Tower of Hanoi Problem, and a set of isomorphs of the very difficut Chinese Ring Puzzle. The experiments determined the relationship between transfer and the difficulty of both the source and target problems. They demonstrate the importance of similarity of representation for transfer, and the primacy of representation over stimulus characteristics of the problems. In addition, the role of transfer in learning the move operators in the problems, and the function of that learning as a substitute for problem exploration were demonstrated. The interaction of the processing demands involved in modifying a skill so that it can be transferred and the demands involved in solving the problem was also explored. The results are presented in detail in the attached technical reports. Keywords:

**20. DISTRIBUTION / AVAILABILITY OF ABSTRACT**
☒ UNCLASSIFIED/UNLIMITED ☐ SAME AS RPT. ☐ DTIC USERS

**21. ABSTRACT SECURITY CLASSIFICATION**
Unclassified

**22a. NAME OF RESPONSIBLE INDIVIDUAL**
Dr. Susan Chipman

**22b. TELEPHONE** *(Include Area Code)*
202-696-4318

**22c. OFFICE SYMBOL**
ONR 1142CS

**DD FORM 1473, 84 MAR** — 83 APR edition may be used until exhausted. All other editions are obsolete.

SECURITY CLASSIFICATION OF THIS PAGE

The Work conducted under ONR contract No. N00014-85-K-0696 constitutes an attempt to achieve an understanding of factors that determine the amount of skill or knowledge that transfers from one problem to another when people solve related problems. In addition to the basic empirical findings that are described herein, the issues that were addressed and the results of the experimental investigations have practical implications for pedagogical practice. While not perhaps the primary focus of the work conducted under this contract, the potential for pedagogical application is substantial. A number of the findings can potentially serve, if not as detailed maps, then at least as broad underpinnings for the design of more effective instructional experiences.

The detailed description of the work that has been done under this contract, and the findings that have been obtained, is presented in the recently completed technical reports that are attached. The attempt here will be to summarize the major findings, and point to areas of convergence, and address some issues that are not included in the pair of attached reports.

The first series of experiments reported on are described in detail in the report entitled "Transfer of Training in Problem Solving" by Kenneth Kotovsky and David Fallside. The work will appear in the forthcoming volume Complex Information Processing: The Impact of Herbert A. Simon, edited by David Klahr and Kenneth Kotovsky, Earlbaum, Englewood Cliffs, New Jersey; in press. There are three primary foci of this set of empirical investigations of transfer. These are 1, the role played by problem similarity in eliciting transfer, 2, the locus of transfer within the problem solving process, and 3, the dominance of representation over stimulus qualities in determining the amount of transfer that will be obtained between pairs of problems.

The starting point for the work described in the first report was a 1985 Cognitive Psychology article entitled "What Makes Problems Hard" by Kenneth Kotovsky, John R. Hayes, and Herbert A. Simon. That article described some findings obtained with isomorphs of the Tower of Hanoi problem, a puzzle like problem that, in its original form, involves the physical manipulation of disks that can be placed on and removed from three pegs according to rules that limit the movement according to the sizes of the disks. The major relevant findings reported in that article were that the different isomorphs varied greatly in difficulty with the solution times of the hardest isomorphs being 16 times longer than those of the easiest isomorphs. The differences in difficulty were shown to be due to the memory load imposed in making moves in the various isomorphs. In addition, it was shown that the subjects' behavior characteristically consisted of two phases, an exploratory phase during which they made no net progress toward the goal, and a final path phase which consisted of a very rapid movement from the start to the goal in the last minute or so of the problem, regardless of which isomorph was being solved. The explanation for the two phase solution process was that the exploratory phase was necessary so that the subjects could practice making moves and thereby automate the application of the problem rules. They were generally not successful at reaching (or even approaching) a solution during this phase of relatively slow move making because making moves was so resource demanding that they were seldom able to plan more than a single move. Once they had achieved the ability to easily make moves and were thus able to plan move pairs, they rapidly reached a solution. Some initial transfer findings in that work were that the problem rules for making moves not only controlled the entree to the final path, but also exerted a degree of control over transfer. Problem isomorphs that had similar move operators yielded larger amounts of transfer than problem isomorphs whose move operators were reversed. This was the case despite the fact that all of the problems shared a common move sequence and external problem space, i.e., were isomorphic.

The current work on transfer[1] started with the hypothesis that the commonality of the move operators as well as the similarity of other problem characteristics was of major importance in transfer. The first set of experiments used an extensive set of problem isomorphs that were computerized versions of the isomorphs used in previous experimentation. The isomorphs were constructed so that their similarity could be varied in a number of ways. Ranked from most to least similar, the problems could be identical, they could have the same basic representation and move operators but have different solution paths, they could have the same basic representations but have reversed move operators, or they could have fundamentally different basic representations. The general finding was that as the problem differences increased the amount of transfer decreased. In addition, the previously noted two phase solution process was very evident. The computerized problem presentation and recording of responses and response latencies allowed for a fine grained analysis of the behavior that confirmed the explanation that the acquisition of "expertise" at making moves immediately preceded the quick movement to a solution that has been labeled

---

[1] Transfer was generally defined as the percentage reduction in the solution time of a problem when it was solved following some other problem as compared to its solution time when it was solved in initial position.

" final path". While the finding of this section of the report, that increased inter-problem similarity results in increased transfer, is not surprising, the demonstration of the effect of particular features of inter-problem similarity on transfer is of interest, as is the replication of the finding of a two-phase solution process.

The second area investigated in these studies was the particular locus of the transfer effect. Having noted that the solution of a range of isomorphs revealed a two-stage process, labelled "exploratory" and "final path", an attempt was made to determine whether transfer was localized in one or the other of the phases. The hypothesis was that transfer might have a function similar to that of problem exploration; the teaching of the problem move operators which then allows for the planning of move pairs. The finding was that transfer selectively reduces exploratory but not final path time. This finding is central to the understanding of transfer that has developed out of this work. Given the localization of the transfer effect in the exploratory phase of problem solving, we further hypothesized that this was likely to result from its having the same effect as problem exploration, the learning of the move operator. We tested this idea by examining the speed with subjects made individual moves and found that the time per move in the exploratory phase of the transfer problem was substantially reduced. The direct implication is that transfer substitutes for problem exploration in teaching problem solvers the move operators, and thus allows them to more quickly reach the final path phase of problem solving.

The third focus of these experiments was on the relative roles played by (a) the similarity in the appearances of the pair of presented problems and (b) the similarity in the manner in which the pair of problems were represented, in the control of transfer. In the experiments directed toward this issue, two problems were presented together with one of two different cover stories. The problem stimuli were again, isomorphs of the Tower of Hanoi problem. These isomorphs involved spheres presented on the screen of a MicroVAX controlled graphics terminal which were interpreted as either moving in depth (in and out of the screen), or as growing larger and smaller in size. The depth or size interpretation were elicited by the cover stories that defined the problems. The major issue investigated was whether the appearance of the problem stimuli, or the representations elicited by the cover stories would play the larger role in determining transfer. The result was that representational similarity was the more important factor. The superficial appearance of the problems was not the controlling factor in whether or not transfer was obtained. Positive transfer was obtained when pairs of problems were represented similarly (two size problem representations or two depth problem representations). The stimulus properties were not the controlling entities. By eliciting representations that were similar, positive transfer could be obtained.

A secondary finding of this series of experiments was that the availability of a representation had an effect on the amount of transfer that was obtained. By varying some of the stimulus qualities of the presented problem we were able to make the size or depth representation more or less easily achieved. The more easily a given representation was evoked, the more available it was a source of transfer to a subsequently presented problem.

In total, this first set of investigations seemed to illuminate a number of issues affecting the transfer of training that can be obtained between problems. The isomorphism of the problems allowed for a large degree of control of the features by which the problems differred or were similar. The findings demonstrated the locus of transfer effects, the factors that elicit transfer, and the crucial importance of representation in controlling transfer.

In an attempt to further our understanding of the interaction of processing limitations, problem difficulty, and transfer, we have begun construction of a computer production system model of a limited capacity problem solver to try to model the effects we have obtained in the above work. The initial results have been encouraging. The model does exhibit some of the final solution path temporal effects that we see so often in our subjects, and also effectively models some of the difficulty differences we see in our problems. It is still in the formative stages, but the initial results are quite encouraging.

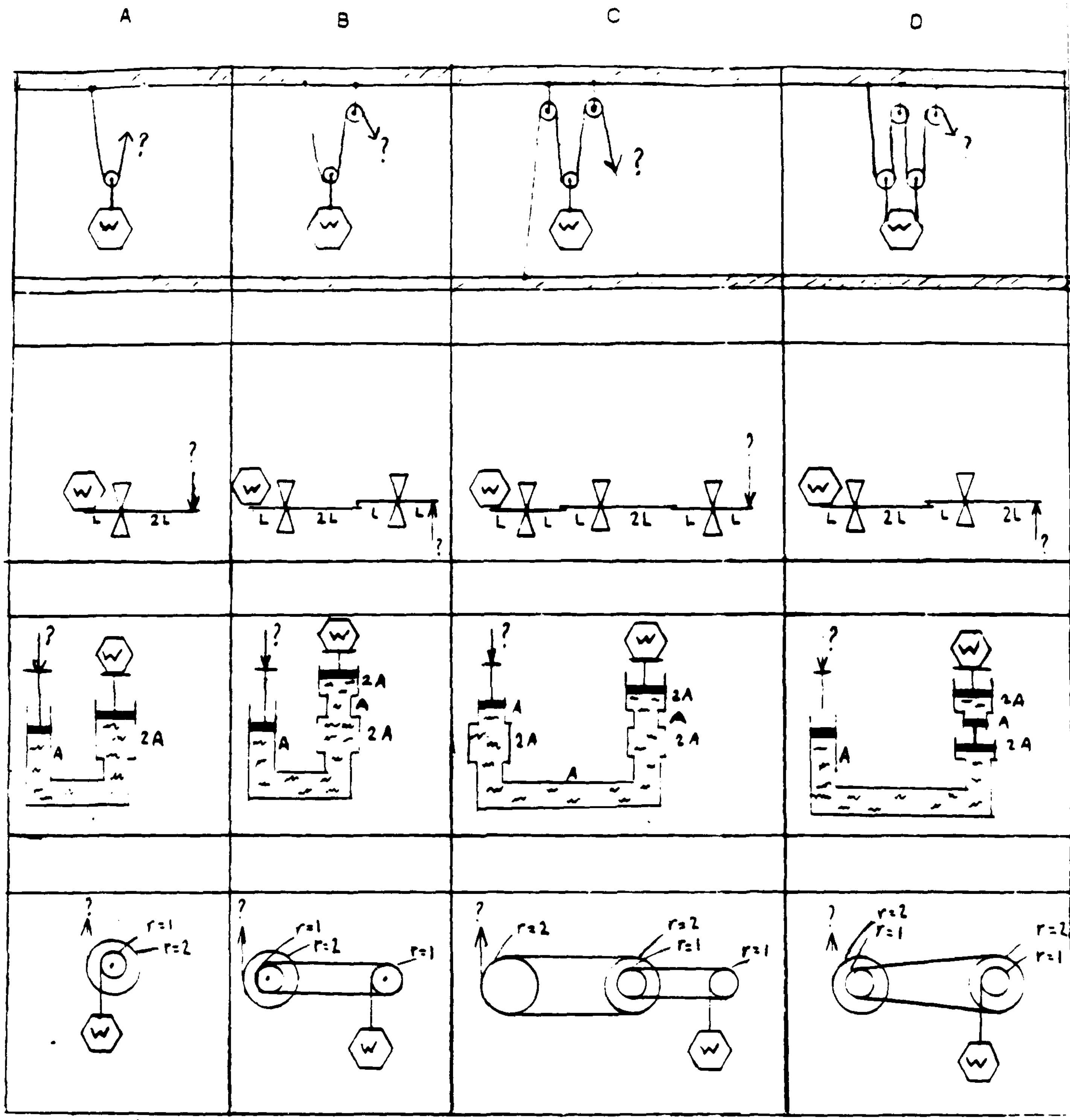
An attempt has begun to extend the above analysis to another problem mileau. A set of physics problem isomorphs was constructed using simple machines as the subject matter. Some examples of the problems are given in the attached Figure.

---------------------------------------

Insert Figure 1 About Here

---------------------------------------

These problems differ from each other in two ways. From left to right in the first three columns they differ in ways that do not affect their mechanical advantage, but rather merely add or subtract elements that at most reverse the direction of the action. The problems in the fourth column do differ in mechanical advantage from those in the other three columns. Moving from row to row, the problems differ in that they involve different machines, but are




A sampling of physics problem isomorphs. The machines within each column are isomorphic to each other. They have equivalent ideal mechanical advantages, realized with equivalent numbers of elements in similar arrangements. Columns B and C show more superficially complex elaborations of the basic (Column A) machine, with the same underlying principle or equivalent ideal mechanical advantage, and the addition of "distractor" elements. Column D depicts a more elaborate machine in which the ideal mechanical advantage is different from the other columns, even though the superficial complexity of the column C machines might seem as great or greater.

similar in having elements that have identical effects on the mechanical advantage of the particular machine. The machines therefore constitute a second set of problem isomorphs that can be used in the investigation of transfer. In a pilot experiment with paper and pencil versions of the problems it was found that they were difficult for subjects, and that little or no transfer was obtained. The ability to solve or not solve a problem seemed to depend more on whether or not the subject had any formal training in physics than on transfer history. Without it, their experience on a prior problem, even though they obtained information about whether they had correctly answered the question about "how hard you would have to pull the rope or press or pull the lever", and were given the correct answer, did not help. The entire set of isomorphs has now been implemented on a MicroVAX so that the subject can not only view the diagram of the machine, but also interact with it. The machine simulations have been programmed so that moving a mouse pointer to the rope or lever (to the position indicated by a question mark in the attached figure) and moving it results in fairly realistic movement of the related machine. We have not yet collected data from this extension of the work on transfer, but are hopeful that the ability to interact with the device will allow for some inter-problem transfer without total reliance on previous physics knowledge.

The second set of experiments to be described reinforce some of the above findings and provide additional information about factors controlling both problem difficulty and transfer between problems. They are described in the second technical report, entitled "Why are Some Problems Really Hard: Explorations in the Problem Space of Difficulty", written with Herbert A. Simon. That report details a number of experiments conducted with isomorphs of the Chinese Ring Puzzle, a very difficult problem involving the manipulation of a physical device consisting of rings that have to be removed from a sliding bar. The problem proved to be virtually unsolvable by our subjects in the two hours of uninterrupted effort they were usually allotted to solve it. To investigate the issues surrounding this problem we constructed isomorphs that were presented on the screen of a computer. These isomorphs preserved all of the features of the external problem space, but introduced various modifications of the move operator by which subjects moved from one state in the problem space to another. In the original puzzle, depicted in Figure 1a of the attached report, a move was made by manipulating a metal ring in such a way that it could be removed from a sliding bar. That movement was usually blocked by a large pin that connected the ring to the bar, and the major task was to discover the nature of a successful move. It is possible to do so, and eventually all subjects did complete some moves. However making moves remained difficult, and the implications of move operator difficulty for both problem solving and for obtaining transfer were the principle foci of the research. In all of the experiments the basic experimental paradigm was a transfer one where the subject was asked to solve two problems, a source problem and a target or transfer problem. We were interested in factors affecting problem difficulty and the reduction of that difficulty via skill transfered from solving the earlier problem. Brief descriptions of the experiments and the findings about problem difficulty are summarized next, followed by a summary of the findings about transfer.

In the first experiment described in the attached technical report, we tested the hypothesis that the source of the great difficulty of the Chinese Puzzle is the move operator. We tested this hypothesis by comparing subject performance on the Chinese Puzzle with their performance on a number of computer isomorphs. We have characterized the major difference between a move in the Chinese Puzzle and a move on one of the computer isomorphs as being that of an analog move in which making a move involves a more or less continuous physical movement of the object which appears capable of being in a large number of possible configurations, versus a digital move, in which a move is made by clicking a mouse pointer on a box containing a ball that, if the move is a legal one, can move out of or into the box. The move in this case has a digital feel to it in that the allowed states are dichotomous: a ball can be either in or out, and the move from one to the other of these states is a single discrete operation. Our first finding was that digitization of the moves made the problem much easier. A problem that in its original form was impossible for our subjects to solve in two and a half hours was routinely solved in 10 to 20 minutes, depending on the particular digital isomorph used. The digital isomorphs were designed to give the subjects different amounts of information about move legality. The least informative, the "No-Info" problem, simply presented the problem and either allowed or did not allow a move to occur based on its legality. A second isomorph, the "Lo-Info" problem, displayed which pair of moves was legal by having the box lids open or close. This had the effect of sharply reducing attempts to make illegal moves. The most informative isomorph, the "All-Info" problem, had a complex series of attachments to the balls that opened and closed the boxes and otherwise blocked the movement of balls that would have otherwise been illegally moved. This isomorph provided a physical rationale for the restrictions on ball movement. The isomorphs are depicted in Figures 1 and 6 of the attached report. The digitization in all of these isomorphs was designed to test whether the move operator was the source of the large difficulty of the Chinese Puzzle. By making the problem relatively easy, it showed that the move operator was the source of difficulty. One implication of this finding, discussed below, is that the move operator should therefore exert a degree of control over transfer.

A second experiment was conducted to provide a further test of the move operator as source of difficulty hypothesis. In this experiment, a strong hint about how to move was given to the subjects. As expected, the hint helped greatly on the Chinese Puzzle, making that problem solvable, and had relatively little effect on the digital isomorphs which had easier move operators. This supported the conclusion that the control of difficulty was in the move operators.

A further argument in support of that conclusion was derived from the shape of the external problem space. The space was linear. If a subject proceeded from the starting position without reversing, they would reach a solution. There was no branching of any kind in the problem space. This sharply contrasts with the usual explanation of problem difficulty being due to exponentially large search spaces in many problems. While there was a large number of moves to reach a solution (a minimum of 21 moves in one version, and 31 in another which differed only in the starting position), the subject had to merely not backtrack in order to solve the problem. This feature of the problem space suggests that even the digital isomorphs were surprisingly difficult. This anomaly also suggested a more direct investigation of the effect of problem space size on the difficulty of attaining a solution in which we compared subject performance on a 21 move problem with their performance on a 31 move problem. We found no difference in difficulty. This experiment directly tested the effect of search space size on difficulty in these problems and found it entirely absent. The thrust of all of the work we report is that the external problem space size is not a factor that controls either the level of difficulty in these problems, nor, as we report below, did the commonality of the problem space guarantee significant transfer.

Transfer effects between the analog and digital problems in this experiment are at best small. Solving a digital isomorph first did not enable the subjects to solve the Chinese Ring Puzzle in the hour and a half to two hours allotted to it. While the number of subjects in this condition was small, there is some indication that prior solution of the Chinese Ring Puzzle did speed the solution of a subsequently presented Lo-Info isomorph. This contrasts with the more typical finding of significant positive transfer between the various digital isomorphs in these experiments. The results thus demonstrate that isomorphism alone is not an adequate predictor of the amount of transfer between two problems.

A second finding is that the amount of information provided on an initial problem is not always a good predictor of transfer, even among digital isomorphs (example: No-Info--Lo-Info) that yield a large amount of transfer. It appears that the cues provided in the Lo-Info isomorph, the depiction of the legal moves, were not useful (and were possibly even counterproductive) for transfer. This contrasts with their marked effect in reducing problem difficulty. The prompts given in the Lo-Info problem may have simply been followed without teaching anything about the conditions for legal moves, in which case the subjects did not gain knowledge they could apply to the second problem, where such prompts were not present. Having been helped through the first problem, subjects, (like students in many situations), might have been at a loss when faced with an unprompted problem. We have previously seen indications of this with the Tower of Hanoi Problem isomorphs as well.

The same crutch-like cue that produced little added transfer when the Lo-Info problem was a source problem strongly facilitated transfer to that problem. We conjecture that this was because the Lo-Info problem was easy enough to allow subjects, while solving it, to think about its relation to the problem they had solved previously. A more surprising related finding was that the Chinese Ring Puzzle, even when not solved, tended to produce positive transfer to the fairly easy Lo-Info problem.

Conversely, the results with the Chinese Ring Puzzle show that when the target problem is difficult, the amount of transfer may be minimal or even negative. The likely explanation is that when problems are dissimilar, the information obtained from the initial problem must be transformed in various ways to be applicable to the transfer problem. The resources required for this transformation are not available while solving a difficult problem.

The major predictor of transfer was not the size or structure of the problem space but the similarity of move operators between the initial problem and the transfer problem. There was a great deal of transfer between the Lo-Info, No-Info, and All-Info problems, but very little transfer between the Chinese Ring Puzzle and any other problem except possibly the Lo-Info condition in transfer position. The transfer results thus parallel the findings about problem difficulty; the determining factor is the nature of the move operator rather than the problem search space. Isomorphism alone is not a guarantor of transfer.

The transfer results parallel the problem difficulty results. We have shown that the difficulty resides largely in the move operator: isomorphs with difficult-to-discover analog move operators are inordinately difficult, while those with moves that are discrete and easily defined are much easier. In these problems, the limited processing resources the subjects initially bring to the problems are consumed by the task of discovering the nature of the move, to the point where they cannot do the planning, placekeeping, or other simple information processing that allows a solution to be found or skill to be transferred to more difficult isomorphs.

In summary, we have conducted a number of experiments on transfer of skill between pairs of problems in two domains, and explored the relationship between transfer and the difficulty of both the source and target problems. The experiments demonstrate the importance of similarity of representation for transfer, the role of transfer in learning the move operators in these problems, the function of that learning in substituting for problem exploration time and thus hastening the time of entree onto the final solution path where the problems are solved with great speed, the interaction of the processing demands involved in transferring skill with those involved in solving the problem, and the lack of effect of the external problem space.

The problem of transfer is an important one. To the extent that learning is to be generalizable, transfer is necessary, even if difficult to achieve. For this reason, if for no other, delineation of the problem characteristics that enhance the probability of its attainment is a worthy goal and one whose achievement the studies reported here attempt to further.